复 合 材 料 学 报　　第24 卷　第6 期　12 月　2007 年
Acta Materiae Compositae Sinica　　Vol. 24　No. 6　December　2007

文章编号: 1000-3851(2007)06-0166-07

受轴压黏弹性层合圆柱壳的极值点蠕变失稳

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摘　要: 基于Donnell 薄壳理论, 采用准弹性方法, 分析了含初始几何缺陷黏弹性层合圆柱壳极值点形式的延迟失稳特征。由轴向缩短量的突然增加定义失稳临界时间, 对玻璃纤维/环氧树脂层合圆柱壳进行了数值计算。结果表明: 圆柱壳存在临界时间趋于无穷的持久临界荷载; 表征延迟失稳程度的瞬时弹性临界荷载与持久临界荷载之差值随着初始几何缺陷的增加而减小; 边界条件、铺设方式对延迟失稳的影响机制可通过对应弹性层合圆柱壳的缺陷敏感性分析来考察。

关键词: 黏弹性; 层合圆柱壳; 极值点; 延迟失稳; 缺陷敏感性
中图分类号: O344.6　　**文献标志码:** A

Creep buckling with limit point type for viscoelastic laminated circular cylindrical shells under axial compression

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Abstract: Based on the Donnel's shell theory, the creep buckling behavior, in the form of limit point, was investigated for viscoelastic laminated circular cylindrical shells under uniformly axial compression with geometrical imperfections. The quasi-elastic approach was applied to the analysis of end-shorting following elapsed time, and the critical time is determined at which the snap-through of end-shorting occurs. The numerical investigation of glass/epoxy laminated circular cylindrical shells was performed. It is shown that there exist durable critical loads which correspond to infinite critical time. The difference between transient critical load and durable critical load, which characterizes the extent of time-dependent buckling, decreases with the increase of imperfection. The mechanism of the influence on the buckling behavior for ply mode, the amplitude of imperfection and boundary conditions as well can be examined by combining the discussion on the sensitivity of imperfection for the corresponding elastic counterpart.

Keywords: viscoelasticity; laminated circular cylindrical shells; limit point; delayed buckling; sensitivity of imperfection

弹性层合圆柱壳静动力稳定特性的研究已取得了丰富的成果。在较早期, Khot[1] 、周承倜[2] 不计边界效应的影响, 通过设定挠曲模式, 分析了轴向受压层合圆柱壳的极值点失稳。Sheinman 等[3-4] 与Goldfeld[5] 分析了具有不同边界条件的层合圆柱壳在轴压作用下的非线性失稳与几何缺陷敏感性。束永平等[6] 考虑横向剪切变形的影响, 采用摄动法研究了反对称正交铺设弹性层合圆柱壳在外压下的屈曲与后屈曲。沈惠申[7-8] 提出了计及边界效应的边界层理论及相应的奇异摄动方法, 分析了复合材料层合圆柱壳在热、力荷载条件下的后屈曲特性。万志敏等[9] 基于几何非线性理论, 研究了角铺设层合圆柱壳在轴向冲击下的非轴对称动力响应与动力失稳。李道奎与周建平[10] 研究了含环向贯穿裂纹的层合圆柱壳在轴压下的稳定性。

在湿热环境下, 聚合物基复合材料呈现明显的黏弹性力学特性, 故聚合物基复合材料层合圆柱壳屈曲失稳的力学行为具有时间相依性。然而, 有关

收稿日期: 2007-03-02; **收修改稿日期:** 2007-05-15
基金项目: 湖南省自然科学基金(05JJ30008)
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黏弹性层合圆柱壳的静动力稳定性的研究报道还比较少见。Chang 等[11] 基于小挠度假设和高阶剪切变形理论，采用准弹性法研究了黏弹性正交铺设层合圆柱壳的蠕变屈曲；彭凡与傅衣铭[12] 基于一阶剪切变形理论，由线性系统动力稳定条件得到正交铺设层合圆柱壳的持久临界荷载。考虑几何非线性后，黏弹性层合圆柱壳的蠕变屈曲及后屈曲行为的分析变得更为复杂，Huang[13] 基于准弹性法对黏弹性层合圆柱壳的极值点失稳进行了定性描述。彭凡等研究了对称铺设层合圆柱壳的非线性蠕变分岔屈曲[14]。Huang[15] 与 Cederbaum[16] 分别对线性与非线性黏弹性层合圆柱曲板的稳定性进行了分析，结果表明，在一定的几何参数和初始几何缺陷条件下，轴向受压层合圆柱曲板会产生极值点形式的延迟失稳。Huang 指出，相比于黏弹性解法，准弹性解法所得结果偏于保守，但当层合圆柱曲板具较大的初始几何缺陷时，2 种解法的结果相差较小。本文中采用准弹性法，对受轴压黏弹性层合圆柱壳的蠕变失稳特征进行分析，结合弹性层合圆柱壳的缺陷敏感性分析，讨论铺设方式、初始几何缺陷大小以及边界条件对层合圆柱壳延迟失稳的影响。

1 控制方程

采用 Voigt 记法，铺设单层在材料主坐标中的应力与应变松弛型本构关系表示为

$$\sigma = E_{ij} \mathrm{d}\varepsilon \quad (i, j = 1, 2, 6) \tag{1}$$

式中：d 表示 Stieltjes 卷积；E_{ij} 为松弛模量。记 S_{ij} 为蠕变柔量，材料的蠕变型本构关系可写为

$$\varepsilon = S_{ij} \mathrm{d}\sigma \quad (i, j = 1, 2, 6) \tag{2}$$

设层合圆柱壳的中面半径、轴向长度和厚度分别为 R、L 和 H，中曲面上直角坐标轴 x 沿轴向，y 沿环向，z 坐标沿径向朝外，柱壳两端沿轴向受均布压荷载 p 作用。设其中曲面挠度为 w，沿轴向与环向的面内位移分别记为 u 和 v，w_0 为初始几何缺陷。层合圆柱壳的中面应变分别为

$$\varepsilon_x^{(0)} = u_{,x} + \frac{1}{2} w_{,x}^2 + w_{,x} w_{0,x},$$

$$\varepsilon_y^{(0)} = v_{,y} + \frac{w}{R} + \frac{1}{2} w_{,y}^2 + w_{,y} w_{0,y},$$

$$\gamma_{xy}^{(0)} = u_{,y} + v_{,x} + w_{,x} w_{,y} + w_{,x} w_{0,y} + w_{0,x} w_{,y}. \tag{3}$$

基于经典层合理论，得物理关系为

$$\begin{matrix} N \\ M \end{matrix} = \begin{matrix} [A_{ij}] & [B_{ij}] \\ [B_{ij}] & [D_{ij}] \end{matrix} \mathrm{d} \begin{matrix} \varepsilon^{(0)} \\ \kappa \end{matrix} \tag{4}$$

式中：$N = (N_x, N_y, N_{xy})^T$ 和 $M = (M_x, M_y, M_{xy})^T$ 分别为薄膜内力与内力矩；$\varepsilon^{(0)} = (\varepsilon_x^{(0)}, \varepsilon_y^{(0)}, \gamma_{xy}^{(0)})^T$，$\kappa = (-w_{,xx}, -w_{,yy}, -2w_{,xy})^T$ 分别为中面应变和中曲面曲率改变；$[A_{ij}]$、$[D_{ij}]$ 和 $[B_{ij}]$ 分别为面内刚度、弯曲刚度以及耦合刚度相对应的矩阵。式(4) 可改写为

$$\begin{matrix} \varepsilon^{(0)} \\ M \end{matrix} = \begin{matrix} [a_{ij}] & -[b_{ij}] \\ [b_{ij}]^T & [d_{ij}] \end{matrix} \mathrm{d} \begin{matrix} N \\ \kappa \end{matrix} \tag{5}$$

其中，$[a_{ij}]$ 为面内蠕变柔量矩阵，在不考虑温度、湿度改变以及老化等因素的条件下，有

$$[a_{ij}] \quad \mathrm{d}[A_{ij}] = I\,H(t) \tag{6}$$

其中：I 是三阶单位矩阵；$H(t)$ 是 Heaveside 函数。式(5) 右边参数矩阵分别为 $[b_{ij}] = [a_{ij}] \quad \mathrm{d}[B_{ij}]$，$[b_{ij}]^T = [B_{ij}]^T \quad \mathrm{d}[a_{ij}]^T$，$[d_{ij}] = [D_{ij}] - ([B_{ij}] \quad \mathrm{d}[a_{ij}]) \quad \mathrm{d}[B_{ij}]$。

准静态下，层合圆柱壳的 Donnell 型平衡方程与变形协调方程为

$$N_{x,x} + N_{xy,y} = 0, \quad N_{xy,x} + N_{y,y} = 0,$$

$$M_{x,xx} + 2M_{xy,xy} + M_{y,yy} - \frac{N_y}{R} + N_x(w_{,xx} + w_{0,xx}) + 2N_{xy}(w_{,xy} + w_{0,xy}) + N_y(w_{,yy} + w_{0,yy}) = 0,$$

$$\varepsilon_{x,xx}^{(0)} + \varepsilon_{y,yy}^{(0)} - 2\gamma_{xy,xy}^{(0)} = \frac{w_{,xx}}{R} + w_{,xy}^2 + 2w_{,xy} w_{0,xy} - w_{,xx} w_{,yy} - w_{,xx} w_{0,yy} - w_{0,xx} w_{,yy}. \tag{7}$$

采用文献[3-5] 给出的第 3 类边界条件，分别考虑圆柱壳两端简支与两端固支，在 $x = 0$，L，有

$$\mathrm{SS3}: w = 0;\ N_x = -p;\ M_x = 0;\ v_{,y} = 0 \tag{8}$$

$$\mathrm{CC3}: w = 0;\ w_{,x} = 0;\ N_x = -p;\ v_{,y} = 0 \tag{9}$$

层合圆柱壳的轴向缩短量由下式计算：

$$\Delta = \frac{1}{2\pi R} \int_0^{2\pi R} \int_0^L u_{,x} \mathrm{d}x\mathrm{d}y \tag{10}$$

本文中采用准弹性法求解，忽略材料的记忆效应，将本构方程与物理关系中的卷积运算近似为乘积运算，则有 $[a_{ij}] = [A_{ij}]^{-1}$，$[b_{ij}] = [a_{ij}][B_{ij}]$，$[d_{ij}] = [D_{ij}] - [B_{ij}][a_{ij}][B_{ij}]$。

引入应力函数 $F(x, y, t)$，且令 $N_x = F_{,yy} - p$，$N_y = F_{,xx}$，$N_{xy} = -F_{,xy}$，式(7) 的前 2 式自动满足，再将式(5) 代入式(7) 的第 3、4 式，得到以挠度和应力函数表示的控制方程。然后定义无量纲参数：$\xi = x/L$，$\eta = y/R$，$\lambda = L/R$，$k_r = L^2/(RH)$，$(W, \omega) = (w, w_0)/H$，$\Phi = F/(A_{220} H^2)$，$\bar{a}_{ij} = A_{ij}/A_{220}$，$\bar{b}_{ij} = B_{ij}/(A_{220} H)$，$(\bar{d}_{ij}, \bar{\bar{d}}_{ij}) = (D_{ij}, d_{ij})/(A_{220} H^2)$，$\bar{\bar{a}}_{ij} = a_{ij} A_{220}$，$\bar{\bar{b}}_{ij} = b_{ij}/H$ $(i, j = 1, 2, 6)$，

$P = pL^2/(A_{220}H^2)$，其中 $A_{220} = A_{22}(0)$。则无量纲化后的控制方程为

$$\begin{aligned}
&\bar{a}_{11}W_{,\xi\xi\xi\xi} + 4\lambda^2\bar{a}_{16}W_{,\xi\xi\xi\eta} + 2\lambda^2(\bar{a}_{12}+2\bar{a}_{66})W_{,\xi\xi\eta\eta} \\
&+ 4\lambda^2\bar{a}_{26}W_{,\xi\eta\eta\eta} + \lambda^4\bar{a}_{22}W_{,\eta\eta\eta\eta} - \bar{a}_{11}\Phi_{,\xi\xi\xi\xi} - \lambda^2(2\bar{a}_{16}-\bar{a}_{26})\Phi_{,\xi\xi\xi\eta} \\
&- \lambda^2(\bar{a}_{11}+\bar{a}_{12}-2\bar{a}_{66})\Phi_{,\xi\xi\eta\eta} - \lambda^2(2\bar{a}_{16}-\bar{a}_{26})\Phi_{,\xi\eta\eta\eta} - \lambda^4\bar{a}_{22}\Phi_{,\eta\eta\eta\eta} + k_r\Phi_{,\xi} \\
&= \lambda^2\Phi_{,\eta}(W_{,\xi\xi}+\bar{w}_{,\xi\xi}) - P(W_{,\xi}+\bar{w}_{,\xi}) \\
&- 2\lambda^2\Phi_{,\xi}(W_{,\xi\eta}+\bar{w}_{,\xi\eta}) + \lambda^2\Phi_{,\xi\xi}(W_{,\eta}+\bar{w}_{,\eta}),
\end{aligned}$$

$$\begin{aligned}
&\bar{b}_{11}W_{,\xi\xi\xi\xi} + \lambda^2(2\bar{b}_{16}-\bar{b}_{61})W_{,\xi\xi\xi\eta} + \lambda^2(\bar{b}_{11}+\bar{b}_{12} \\
&- 2\bar{b}_{66})W_{,\xi\xi\eta\eta} + \lambda^2(2\bar{b}_{16}-\bar{b}_{62})W_{,\xi\eta\eta\eta} + \lambda^4\bar{b}_{12}W_{,\eta\eta\eta\eta} \\
&- k_rW_{,\xi\xi} + \bar{b}_{12}\Phi_{,\xi\xi\xi\xi} - 2\lambda^2\bar{b}_{16}\Phi_{,\xi\xi\xi\eta} + \lambda^2(2\bar{b}_{12} \\
&+ \bar{b}_{66})\Phi_{,\xi\xi\eta\eta} - 2\lambda^2\bar{b}_{26}\Phi_{,\xi\eta\eta\eta} + \lambda^4\bar{b}_{11}\Phi_{,\eta\eta\eta\eta} \\
&= \lambda^2W^2_{,\xi\eta} + 2\lambda^2W_{,\xi\eta}\bar{w}_{,\xi\eta} - \lambda^2W_{,\xi\xi}W_{,\eta} \\
&- \lambda^2W_{,\xi\xi}\bar{w}_{,\eta} - \lambda^2W_{,\eta}\bar{w}_{,\xi\xi}. \qquad (11)
\end{aligned}$$

由式(8)和式(9)，在 $\xi=0,1$，以挠度与应力函数表示的无量纲化边界条件分别为

SS3：$W=0$，$\Phi_{,\eta}=0$，

$$\begin{aligned}
&\bar{b}_{11}P - \bar{b}_{11}\Phi_{,\xi\xi} + \bar{b}_{16}\lambda\Phi_{,\xi\eta} + \bar{a}_{11}W_{,\xi\xi} + \bar{a}_{12}\lambda^2W_{,\eta} \\
&+ 2\bar{a}_{16}\lambda W_{,\xi\eta} = 0,
\end{aligned}$$

$$\begin{aligned}
&\bar{b}_{12}\Phi_{,\xi\xi} - \bar{b}_{12}P - \bar{b}_{16}\lambda\Phi_{,\xi\eta} + \bar{b}_{11}W_{,\xi\xi} + \bar{b}_{12}\lambda^2W_{,\eta} \\
&+ 2\bar{b}_{16}\lambda W_{,\xi\eta} - \frac{1}{2}\lambda^2W^2_{,\eta} - \lambda^2W_{,\eta}\bar{w}_{,\eta} = 0. \qquad (12)
\end{aligned}$$

CC3：$W=0$，$W_{,\xi}=0$，$\Phi_{,\eta}=0$，

$$\begin{aligned}
&\bar{b}_{12}\Phi_{,\xi\xi} - \bar{b}_{12}P - \bar{b}_{16}\lambda\Phi_{,\xi\eta} + \bar{b}_{11}W_{,\xi\xi} + \bar{b}_{12}\lambda^2W_{,\eta} \\
&+ 2\bar{b}_{16}\lambda W_{,\xi\eta} - \frac{1}{2}\lambda^2W^2_{,\eta} - \lambda^2W_{,\eta}\bar{w}_{,\eta} = 0. \qquad (13)
\end{aligned}$$

需指出的是，式(11)～(13)中的刚度和柔度参数与时间相关。

2　求解方法

弹性层合圆柱壳的非线性失稳为极值点失稳[2]。对于黏弹性层合圆柱壳，若所加轴压小于对应的弹性临界荷载，在发生失稳前，将经历与时间相关的蠕变变形。本文中假设黏弹性层合圆柱壳蠕变失稳的环向波形与弹性失稳时环向波形的形式相同。虽然环向波形的模式不变，但经历了前屈曲的蠕变变形阶段，其组合方式发生了改变，也正是因为这种改变，引起黏弹性层合圆柱壳与时间相关的极值点失稳。为此，将挠度和应力函数表示为如下分离变量形式[3]：

$$\begin{aligned}
W(\xi,\eta,t) &= W_0(\xi,t) + \sum_{i=1}^{k}[W_{1i}(\xi,t)\cos(in\pi\eta) \\
&+ W_{2i}(\xi,t)\sin(in\pi\eta)],
\end{aligned}$$

$$\begin{aligned}
\Phi(\xi,\eta,t) &= \Phi_0(\xi,t) + \sum_{i=1}^{2k}[\Phi_{1i}(\xi,t)\cos(in\pi\eta) \\
&+ \Phi_{2i}(\xi,t)\sin(in\pi\eta)],
\end{aligned}$$

$$\begin{aligned}
\bar{w}(\xi,\eta) &= \bar{w}_0(\xi) + \sum_{i=1}^{k}[\bar{w}_{1i}(\xi)\cos(in\pi\eta) \\
&+ \bar{w}_{2i}(\xi)\sin(in\pi\eta)]. \qquad (14)
\end{aligned}$$

式中：n 为弹性失稳环向波数，由对应的弹性极值点失稳分析确定；i 为求和指标。取 $k=1$，将式(14)代入式(11)的第2式，比较两边相同三角函数项的系数，可得如下表达式：

$$\begin{aligned}
&\bar{b}_{11}W_{0,\xi\xi\xi\xi} - k_rW_{0,\xi} + \bar{a}_{12}\Phi_{0,\xi\xi\xi\xi} = \lambda n^2\left(\frac{1}{2}W^2_{11,\xi}\right. \\
&+ \frac{1}{2}W^2_{21,\xi} + W_{11,\xi}\bar{w}_{11,\xi} + W_{21,\xi}\bar{w}_{21,\xi} + \frac{1}{2}W_{11,\xi\xi}W_{11} \\
&+ \frac{1}{2}W_{21,\xi\xi}W_{21} + \frac{1}{2}W_{11,\xi\xi}\bar{w}_{11} + \frac{1}{2}W_{21,\xi\xi}\bar{w}_{21} \\
&\left.+ \frac{1}{2}W_{11}\bar{w}_{11,\xi\xi} + \frac{1}{2}W_{21}\bar{w}_{21,\xi\xi}\right) \qquad (15a)
\end{aligned}$$

$$\begin{aligned}
&\bar{b}_{11}W_{11,\xi\xi\xi\xi} + \lambda n(2\bar{b}_{16}-\bar{b}_{61})W_{21,\xi\xi\xi} - (\lambda n)^2(\bar{b}_{11}+\bar{b}_{12} \\
&- 2\bar{b}_{66})W_{11,\xi\xi} - (\lambda n)^3(2\bar{b}_{16}-\bar{b}_{62})W_{21,\xi} \\
&+ (\lambda n)^4\bar{b}_{12}W_{11} - k_rW_{11,\xi\xi} + \bar{a}_{12}\Phi_{11,\xi\xi\xi\xi} - 2\lambda n\bar{a}_{16}\Phi_{11,\xi\xi\xi} \\
&- (\lambda n)^2(2\bar{a}_{12}+\bar{a}_{66})\Phi_{11,\xi\xi} + 2(\lambda n)^3\bar{a}_{16}\Phi_{11,\xi} \\
&+ (\lambda n)^4\bar{a}_{11}\Phi_{11} = (\lambda n)^2(W_{0,\xi}W_{11} + W_{0,\xi}\bar{w}_{11} \\
&+ \bar{w}_{0,\xi}W_{11}) \qquad (15b)
\end{aligned}$$

$$\begin{aligned}
&\bar{b}_{11}W_{21,\xi\xi\xi\xi} - \lambda n(2\bar{b}_{16}-\bar{b}_{61})W_{11,\xi\xi\xi} - (\lambda n)^2(\bar{b}_{11}+\bar{b}_{12} \\
&- 2\bar{b}_{66})W_{21,\xi\xi} + (\lambda n)^3(2\bar{b}_{16}-\bar{b}_{62})W_{11,\xi} \\
&+ (\lambda n)^4\bar{b}_{12}W_{21} - k_rW_{21,\xi\xi} + \bar{a}_{12}\Phi_{21,\xi\xi\xi\xi} + 2\lambda n\bar{a}_{16}\Phi_{21,\xi\xi\xi} \\
&- (\lambda n)^2(2\bar{a}_{12}+\bar{a}_{66})\Phi_{21,\xi\xi} - 2(\lambda n)^3\bar{a}_{16}\Phi_{21,\xi} \\
&+ (\lambda n)^4\bar{a}_{11}\Phi_{21} = (\lambda n)^2(W_{0,\xi}W_{21} + W_{0,\xi}\bar{w}_{21} \\
&+ \bar{w}_{0,\xi}W_{21}) \qquad (15c)
\end{aligned}$$

$$\begin{aligned}
&\bar{a}_{12}\Phi_{12,\xi\xi\xi\xi} - 4\lambda n\bar{a}_{16}\Phi_{12,\xi\xi\xi} - 4(\lambda n)^2(2\bar{a}_{12}+\bar{a}_{66})\Phi_{12,\xi\xi} \\
&+ 16(\lambda n)^3\bar{a}_{16}\Phi_{12,\xi} + 16(\lambda n)^4\bar{a}_{11}\Phi_{12} \\
&= \frac{1}{2}(\lambda n)^2(-W^2_{11,\xi} + W^2_{21,\xi} - 2W_{11,\xi}\bar{w}_{11,\xi} \\
&+ 2W_{21,\xi}\bar{w}_{21,\xi} + W_{11,\xi\xi}W_{11} - W_{21,\xi\xi}W_{21} + W_{11,\xi\xi}\bar{w}_{11} \\
&- W_{21,\xi\xi}\bar{w}_{21} + W_{11}\bar{w}_{11,\xi\xi} - W_{21}\bar{w}_{21,\xi\xi}) \qquad (15d)
\end{aligned}$$

$$\begin{aligned}
&\bar{a}_{12}\Phi_{22,\xi\xi\xi\xi} + 4\lambda n\bar{a}_{16}\Phi_{22,\xi\xi\xi} - 4(\lambda n)^2(2\bar{a}_{12}+\bar{a}_{66})\Phi_{22,\xi\xi} \\
&- 16(\lambda n)^3\bar{a}_{16}\Phi_{22,\xi} + 16(\lambda n)^4\bar{a}_{11}\Phi_{22} \\
&= \frac{1}{2}(\lambda n)^2(-2W_{11,\xi}W_{21,\xi} - 2W_{11,\xi}\bar{w}_{21,\xi} \\
&- 2W_{21,\xi}\bar{w}_{11,\xi} + W_{11,\xi\xi}W_{21} + W_{21,\xi\xi}W_{11} + W_{11,\xi\xi}\bar{w}_{21} \\
&+ W_{21,\xi\xi}\bar{w}_{11} + W_{21}\bar{w}_{11,\xi\xi} + W_{11}\bar{w}_{21,\xi\xi}) \qquad (15e)
\end{aligned}$$

将式(14)代入式(11)的第1式，取权函数分别为1、$\cos(n\eta)$、$\sin(n\eta)$，沿圆柱壳环向进行 Galer-

kin 积分, 导出如下方程:

$$\bar{D}_{11} W_{0,xxxx} - \bar{B}_{11} \Phi_{0,xxxx} + k_r \Phi_{0,xx} = -\frac{(\lambda n)^2}{2}[\Phi_{11}(W_{11,xx} + w_{11,xx}) + \Phi_{11}(W_{21,xx} + w_{21,xx}) + 2\Phi_{11,x}(W_{11,x} + w_{11,x}) + 2\Phi_{11,x}(W_{21,x} + w_{21,x}) + \Phi_{11,xx}(W_{11} + w_{11}) + \Phi_{11,xx}(W_{21} + w_{21})] - P(W_{0,xx} + w_{0,xx}) \tag{16a}$$

$$\bar{D}_{11} W_{11,xxxx} + 4\lambda n \bar{D}_{16} W_{21,xxx} - 2(\lambda n)^2(\bar{D}_{12} + 2\bar{D}_{66}) W_{11,xx} - 4(\lambda n)^3 \bar{D}_{16} W_{21,xx} + (\lambda n)^4 \bar{D}_{22} W_{11} - \bar{B}_{11} \Phi_{11,xxxx} - \lambda n(2\bar{B}_{16} - \bar{B}_{26}) \Phi_{11,xxx} + (\lambda n)^2(\bar{B}_{11} + \bar{B}_{22} - 2\bar{B}_{66}) \Phi_{11,xx} + (\lambda n)^3(2\bar{B}_{16} - \bar{B}_{26}) \Phi_{11,x} + (\lambda n)^4 \bar{B}_{22} \Phi_{11} + k_r \Phi_{11,xx} = -(\lambda n)^2[\Phi_{11}(W_{0,xx} + w_{0,xx}) + 2\Phi_{12}(W_{11,xx} + w_{11,xx}) + 2\Phi_{12}(W_{21,xx} + w_{21,xx}) + 2\Phi_{12,x}(W_{11,x} + w_{11,x}) + 2\Phi_{12,x}(W_{21,x} + w_{21,x}) + \Phi_{12,xx}(W_{11} + w_{11}) + \frac{1}{2}\Phi_{12,xx}(W_{11} + w_{11}) + \frac{1}{2}\Phi_{12,xx}(W_{21} + w_{21})] - P(W_{11,xx} + w_{11,xx}) \tag{16b}$$

$$\bar{D}_{11} W_{21,xxxx} - 4\lambda n \bar{D}_{16} W_{11,xxx} - 2(\lambda n)^2(\bar{D}_{12} + 2\bar{D}_{66}) W_{21,xx} + 4(\lambda n)^3 \bar{D}_{16} W_{11,xx} + (\lambda n)^4 \bar{D}_{22} W_{21} - \bar{B}_{11} \Phi_{21,xxxx} + \lambda n(2\bar{B}_{16} - \bar{B}_{26}) \Phi_{21,xxx} + (\lambda n)^2(\bar{B}_{11} + \bar{B}_{22} - 2\bar{B}_{66}) \Phi_{21,xx} - (\lambda n)^3(2\bar{B}_{16} - \bar{B}_{26}) \Phi_{21,x} + (\lambda n)^4 \bar{B}_{22} \Phi_{21} + k_r \Phi_{21,xx} = -(\lambda n)^2[\Phi_{21}(W_{0,xx} + w_{0,xx}) + 2\Phi_{12}(W_{11,xx} + w_{11,xx}) - 2\Phi_{12}(W_{21,xx} + w_{21,xx}) + 2\Phi_{12,x}(W_{11,x} + w_{11,x}) - 2\Phi_{12,x}(W_{21,x} + w_{21,x}) + \Phi_{12,xx}(W_{11} + w_{11}) + \frac{1}{2}\Phi_{12,xx}(W_{11} + w_{11}) - \frac{1}{2}\Phi_{12,xx}(W_{21} + w_{21})] - P(W_{21,xx} + w_{21,xx}) \tag{16c}$$

将边界条件也沿环向作 Galerkin 积分, 结合式(15) 与式(16), 得到非线性常微分型方程组以及相应的边界条件, 且其中的系数与时间相关。将该定解问题依中心差分格式沿轴向坐标离散, 得到非线性代数方程组, 首先求得 $t = 0^+$ 所对应的弹性临界轴压, 且确定 n。然后, 给定压屈载荷, 在每一时间步进行迭代求解, 其中的初始迭代值取前一时间步的结果。具体计算时, 差分节点数统一取为 50。

3 数值算例与讨论

设初始几何缺陷形式为[3]

$$w_0(\xi, \eta, t) = W_{G0} \sin(\pi\xi) \cos(n\eta) \tag{17}$$

式中 W_{G0} 为初始几何缺陷的幅值。首先就弹性临界荷载进行验证性计算, 取圆柱壳两端简支, 4 层等厚铺设, 几何参数为 H = 0.54 mm, R = 190.5 mm, L/R = 2。考虑 Boron fiber/epoxy (AVCO 5505) 材料, 其模量参数为 E_1 = 206.850 GPa, E_2 = 18.617 GPa, E_6 = 4.482 GPa, ν_{12} = 0.21。对应式(1), 则有 $E_{11}(0)$ = 207.674 GPa, $E_{12}(0)$ = 3.925 GPa, $E_{22}(0)$ = 18.691 GPa, $E_{66}(0)$ = 4.482 GPa。分别取对称与反对称铺设方式, 表 1 给出了本文与文献[3] 的计算结果, 其中 p_{cr} 为极值点失稳临界轴压, 与初始几何缺陷幅值有关; p_{cr1} 为基于小挠度假设得到的经典屈曲荷载。由该表可见, 本文关于极值点失稳的临界轴压的计算结果与文献[3] 的基本接近, 对应于表中所示的 4 层对称铺设与反对称铺设方式, 层合圆柱壳的缺陷敏感性是接近相同的。

考虑 Glass fiber/epoxy 复合材料, 取 3 层等厚铺设, 圆柱壳的铺设方式分别为对称铺设[20°/0/20°] 与反对称铺设[70°/0/ - 70°], 各表示为铺设方式 I 与铺设方式 II。类似于文献[1], 圆柱壳的几何参数取为 H = 0.9252 mm, R = 154.2 mm, L/R = 2; 而初始缺陷模式仍为式(17) 所描述。取铺设单层的蠕变柔量为[17]

$$S_{11} = 0.0267 S_0, \quad S_{12} = -0.00826 S_0,$$

$$S_{22} = 0.0942(1 + 0.0846 t^{0.27}) S_0,$$

表 1 Boron/epoxy (AVCO 5505) 层合圆柱壳的弹性临界载荷

Table 1 Elastical critical loads of AVCO 5505 composite laminated circular cylindrical shells

		Classic buckling load p_{cr1}/(kN·m^{-1})	Critical load p_{cr}/(kN·m^{-1})			
			W_{G0} = 0.05	W_{G0} = 0.5	W_{G0} = 1	W_{G0} = 2
[45°/-45°/-45°/45°]	Ref[3]	—	25.6	23.8	21.5	17.2
	This paper	25.1	24.5	23.1	21.7	16.5
[45°/-45°/45°/-45°]	Ref[3]	—	24.3	22.8	20.8	16.2
	This paper	25.5	24.7	23.3	22.0	16.6

$S_{66} = 0.233(1 + 0.151t^{0.27})S_0$. (18)

式中: $S_0 = 1\ \mathrm{GPa}^{-1}$; t 为时间, h。表2 给出了两端简支层合圆柱壳在3 种几何初始缺陷下的弹性临界轴压值。可见, 3 层反对称铺设时, 圆柱壳的弹性临界轴压值下降更多, 说明其对几何缺陷更加敏感。进一步比较表1, 可知弹性层合圆柱壳的缺陷敏感性是与铺设方式及铺设角密切相关的。

表2　玻璃纤维/ 环氧树脂圆柱壳弹性临界载荷

Table 2　Elastic critical loads of glass fiber/epoxy composite laminated circular cylindrical shells

	Classic buckling load p_{cr1}/ (kN·m^{-1})	Critical load p_{cr}/ (kN·m^{-1})		
		$W_{G0} = 0.05$	$W_{G0} = 0.1$	$W_{G0} = 0.2$
I :[20°/0°/20°]	46.6	29.1	24.4	23.3
II:[70°/0°/-70°]	50.7	31.7	21.6	20.9

下面分析黏弹性层合圆柱壳与时间相关的极值点失稳。图1 给出了初始几何缺陷幅值 $W_{G0} = 0.05$ 时, 两端简支反对称铺设圆柱壳在不同轴压下的轴向缩端与时间的关系。由图可见, 当轴压为 p = 22.83 kN/m 时, 圆柱壳在经历弹性加载后, 其轴向缩端随时间缓慢增加直至某一稳态值; 而当 p = 25.37 kN/m 时, 轴向缩短增加到一定程度时, 在某一时刻发生突变, 表明此时圆柱壳在当前压屈荷载作用下出现极值点形式的延迟失稳, 而对应时刻称为临界失稳时间; 若压屈荷载增加到 p = 27.91 kN/m 时, 圆柱壳极值点失稳的临界时间缩短。两端简支时, 圆柱壳在 I、II 2 组铺设方式下的压屈荷载和临界时间的关系示于图2。图中, t = 0 对应圆柱壳极值点失稳的“瞬时弹性临界荷载”, 当临界时间增加时, 压屈荷载减小, 且趋于一定值, 后者就是所谓“持久临界荷载”。瞬时弹性临界荷载与持久临界荷载的差值反映了延迟失稳的程度。从图2 中可见, 对3 层对称铺设, 当 W_{G0} 从0.01 增加到0.05 时, 两种临界荷载的差值变小, 尤其是当 $W_{G0} = 0.05$ 时, 延迟失稳现象不明显。而对3 层反对称铺设II, 当 W_{G0} 从0.05 增加到0.1 时, 2 种临界荷载的差值从大约8.1 kN/m 下降到约4.8 kN/m, 可见, 当初始几何缺陷增大, 延迟失稳程度减弱。进一步观察可知, 在 W_{G0} 同为0.05 时, 相比于对称铺设I, 反对称铺设II的延迟失稳现象更为明显。

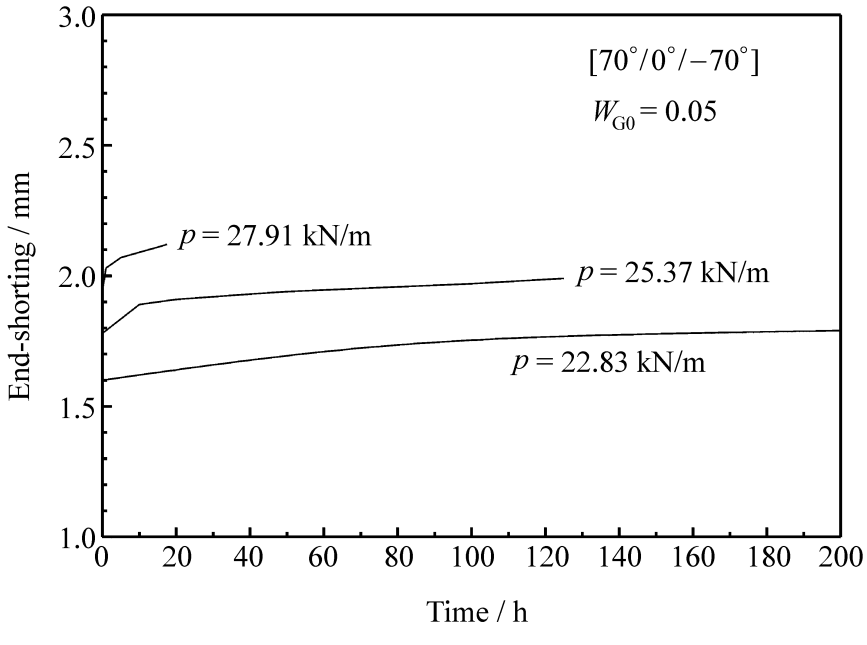

图1　两端简支反对称铺设层合圆柱壳的轴向缩端与时间的关系

Fig.1　End shorting vs time for anti-symmetrically laminated circular cylindrical shells with SS3 ends

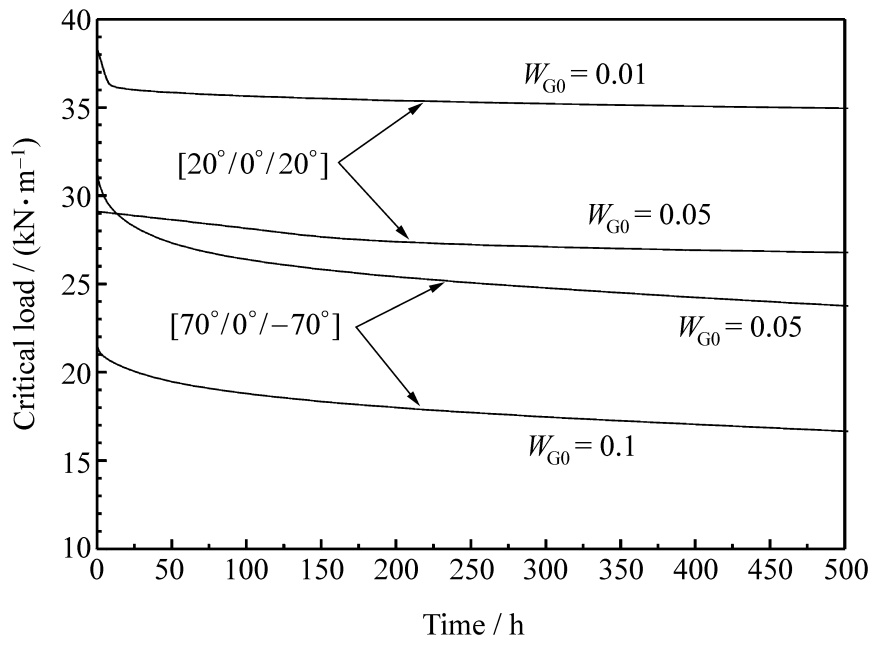

图2　两端简支对称铺设层合圆柱壳的压屈荷载与临界时间的关系

Fig.2　Buckling loads vs critical time for symmetrically laminated circular cylindrical shells with SS3 ends

上述结果可通过分析弹性临界荷载与几何初始缺陷幅值的关系来说明, 如图3 中的实线所示两端简支的情形。当 W_{G0} 小于0.07 时, 反对称铺设II的弹性临界荷载高于对称铺设I 的结果; 而当 W_{G0} 大于0.07 时, 则相反。考虑到对称铺设与反对称铺设条件下, 圆柱壳的经典屈曲轴压分别为46.6 kN/m 和50.7 kN/m(参见表2) , 可见反对称铺设时, 层合圆柱壳的弹性极值点失稳对初始缺陷更为敏感; 从而在黏弹性情形下, 给定初始几何缺陷幅值, 相对于对称铺设I, 反对称铺设II的延迟失稳更为明显。从图3 还可以进一步看出, 针对反对称铺设情形, 在 W_{G0} 的较小范围内, 两端固支时的缺陷敏感

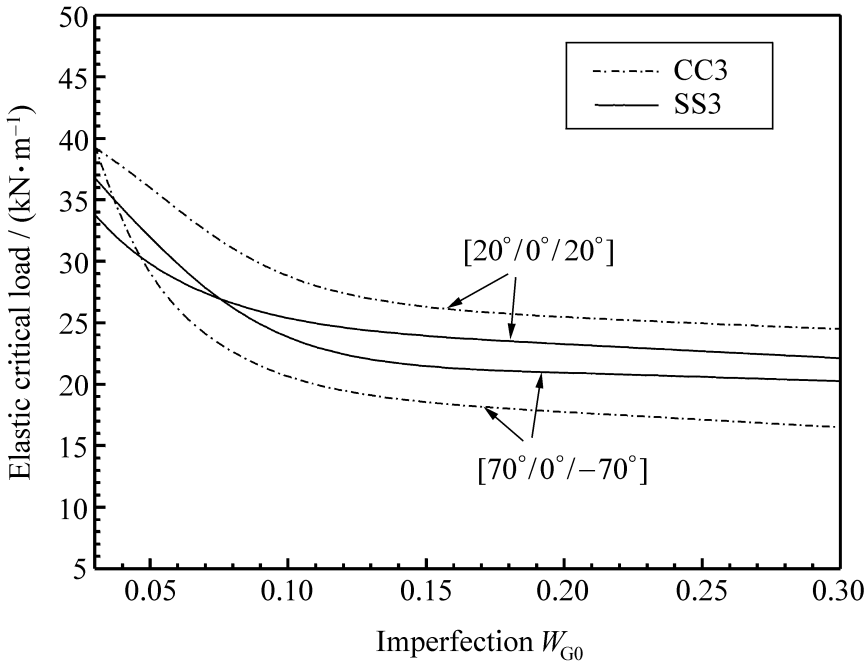

图3 弹性临界荷载与初始几何缺陷的关系

Fig.3 Elastic critical loads vs geometrical imperfection

性较两端简支时的高, 且在 $W_{G0} > 0.03$ 时, 后者的弹性临界荷载高于前者, 而在 W_{G0} 大于0.05 后, 弹性临界荷载随缺陷幅值的下降速度基本相同。取 $W_{G0} = 0.05$, 对应2 组边界条件的压屈荷载- 临界时间曲线示于图4(a) 。可见, 两端固支时的压屈荷载 临界时间曲线位于下方, 且2 组结果关于临界时间的变化趋势也接近相同, 这与图3 所给出的解释一致。

置 $W_{G0} = 0.01$, 考虑对称铺设Ⅰ, 则情形与图4(a) 结果相反, 如图4(b) 所示。两端固支时的压屈荷载- 临界时间曲线位于简支时相应曲线的上方, 后者的弹性临界荷载与持久临界荷载差值高于前者, 这一现象也可同样从图3 的对应曲线来说明。计算结果表明: 当 $W_{G0} = 0.01$ 时, 简支与固支情形下的弹性临界荷载基本相同, 约为38.4 kN/m; 而在 $0.01 < W_{G0} < 0.1$ 的范围内, 简支的弹性临界荷载明显小于固支的结果, 说明在此范围内, 简支比固支的缺陷敏感性高。这是因为对称铺设时, 不存在拉弯耦合, 固支边界更强地抑制了圆柱壳的弯曲, 从而增加了结构刚度, 故从图4(b) 中可观察到, 简支边界的延迟失稳曲线呈现急剧下降的趋势。从以上讨论可知, 黏弹性层合圆柱壳极值点形式的延迟失稳的程度与铺设方式、初始几何缺陷以及边界条件相关, 可以从相应弹性层合圆柱壳的缺陷敏感性进行分析。严格说来, 层合圆柱壳失稳前的蠕变变形应由积分微分型的控制方程求解给出, 即由黏弹性解法得到。本文中采用准弹性法近似求解, 相应给定时间, 得出的临界轴压小于黏弹性解法所给出的结果[15] 。这种数值上的差别还有待于今后的进一步分析, 但对本文中关于层合圆柱壳极值点形式的蠕变失稳特征的分析不会有明显影响。

4 结 论

本文中基于Donnell 薄壳理论, 采用准弹性方法, 分析了具有初始几何缺陷黏弹性层合圆柱壳极值点形式的延迟失稳特征。由轴向缩短量的突然增加来得到失稳临界时间, 对玻璃纤维/环氧树脂层合圆柱壳进行了数值计算, 结果表明: (1) 黏弹性层合圆柱壳存在临界时间趋于无穷的持久临界载荷; (2) 延迟失稳现象随着初始缺陷的增加程度减小, 即弹性临界荷载与持久临界荷载的差值变小, 这种变化规律与铺设状况无关; (3) 边界条件、铺设方式对延迟失稳的影响机制可通过弹性层合圆柱

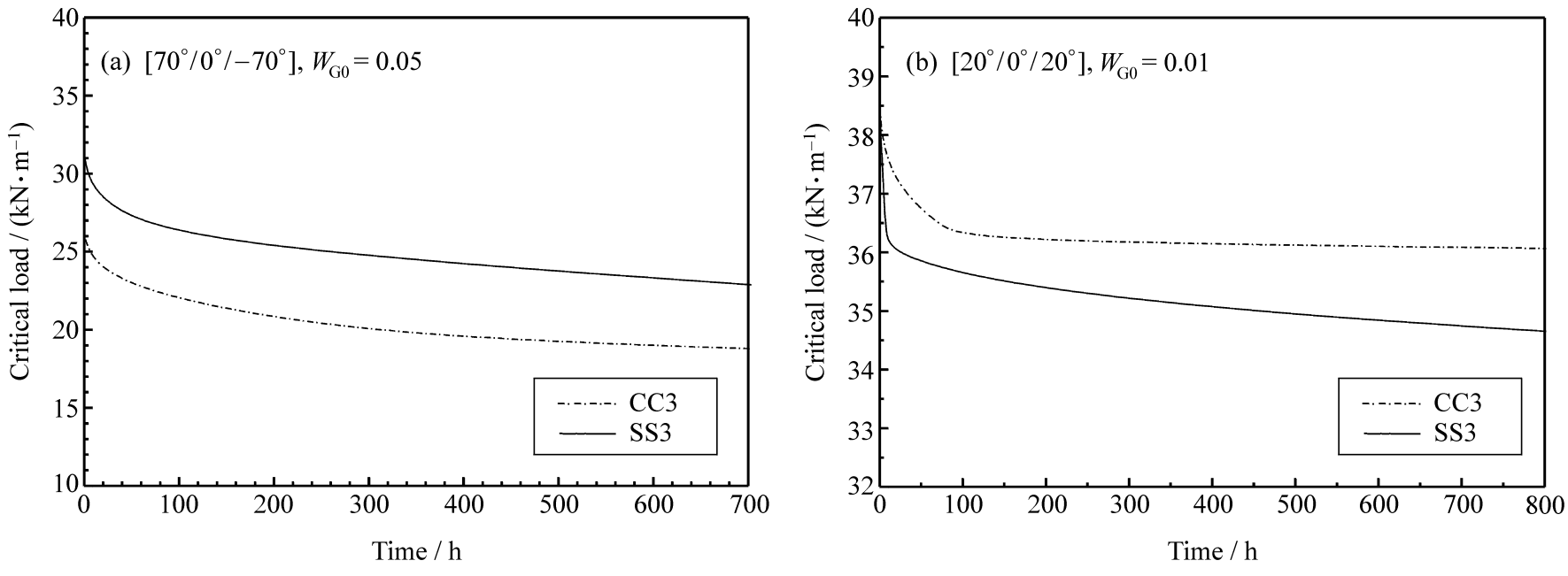

图4 反对称与对称铺设层合圆柱壳的压屈荷载与临界时间的关系

Fig.4 Buckling loads vs critical time for anti-symmetrically and symmetrically laminated circular cylindrical shells

壳的缺陷敏感性分析来考察。

参考文献:

[1] Khot N S. Buckling and postbuckling behavior of composite cylindrical shells under axial compression [J]. AIAA Journal, 1970, 8(2): 229-235.

[2] 周承倜. 复合材料圆柱壳理论的进展[C] // 罗祖道, 王震鸣. 复合材料力学进展. 北京: 北京大学出版社, 1992: 261-295.
Zhou Chengti. Advance in research of stability for composite circular cylindrical shells [C] // Luo Zudao, Wang Zhenming, ed. Advance of Composite Mechanics. Beijing: Peking University Press, 1992: 261-295.

[3] Sheinmen I, Shaw D, Simitses G J. Nonlinear analysis of axially-loaded laminated cylindrical shells [J]. Computers & Structures, 1983, 36(1/4): 131-137.

[4] Sheinmen I, Goldfeld Y. Imperfection sensitivity of laminated cylindrical shells according to different shell theories [J]. Journal of Engineering Mechanics, 2003, 129(8): 1048-1063.

[5] Goldfeld Y. The influence of the stiffness coefficients on the imperfection sensitivity of laminated cylindrical shells [J]. Composite Structure, 2004, 64(2): 243-247.

[6] 束永平, 张建武, 吴金松. 反对称正交铺层剪切圆柱壳在外压下的屈曲与后屈曲[J]. 复合材料学报, 1999, 16(4): 121-129.
Shu Yongping, Zhang Jianwu, Wu Jinsong. Buckling and postbuckling of antisymmetrically laminated cross-ply shear-deformable cylindrical shells under external pressure [J]. Acta Materiae Compositae Sinica, 1999, 16(4): 121-129.

[7] 沈惠申. 板壳后屈曲行为[M]. 上海: 上海科学技术出版社, 2002: 302-382.
Shen Huisheng. Postbuckling behavior of plates and shells [M]. Shanghai: Shanghai Science and Technology Publishing House, 2002: 302-382.

[8] Shen H S. Hygrothermal effects on the post-buckling of composite laminated cylindrical shells [J]. Composite Science and Technology, 2000, 60(8): 1227-1240.

[9] 万志敏, 杜星文, 谢志明. 玻璃/环氧圆柱壳的冲击响应与稳定性分析[J]. 复合材料学报, 2001, 18(4): 82-86.
Wan Zhimin, Du Xingwen, Xie Zhiming. Impact response and stability analysis of glass-epoxy cylindrical shells [J]. Acta Materiae Compositae Sinica, 2001, 18(4): 82-86.

[10] 李道奎, 周建平. 含环向贯穿脱层的轴压圆柱层壳屈曲分析 I: 基本方程与定解条件[J]. 复合材料学报, 2002, 19(1): 74-79.
Li Daokui, Zhou Jianping. Buckling of cylindrical shells with circumferential delamination I: Basic equations and solution-determining conditions [J]. Acta Materiae Compositae Sinica, 2002, 19(1): 74-79.

[11] hang J S, Cheng C H. Viscoelastic buckling of cross ply laminated cylindrical circular shells based on a high order displacement field [J]. Chinese Journal of Mechanics, 1991, 7(2): 99-107.

[12] 彭　凡, 傅衣铭. 黏弹性圆柱壳在轴向恒压下的动力稳定性[J]. 工程力学学报, 2002, 19(6): 49-53.
Peng Fan, Fu Yiming. Dynamic stability of viscoelastic cylindrical shells under constant axial compression [J]. Engineering Mechanics, 2002, 19(6): 49-53.

[13] uang N N. Viscoelastic buckling and postbuckling of circular cylindrical laminated shells in hygrothermal environment [J]. Journal of Marine Science and Technology, 1994, 2(1): 9-16.

[14] 彭　凡, 向　红, 傅衣铭. 黏弹性层合圆柱壳的蠕变分岔屈曲[J]. 力学学报, 2006, 38(6): 792-798.
Peng Fan, Xiang Hong, Fu Yiming. Bifurcation creep buckling of circular cylindrical shells [J]. Chinese Journal of Theoretical and Applied Mechanics, 2006, 38(6): 792-798.

[15] uang N N. Creep deflection of viscoelastic laminated cylindrical panels with initial deflection under axial compression [J]. Composites Part B, 1999, 30(2): 145-156.

[16] ederbaum G, Touati D. Postbuckling analysis of imperfect non-linear viscoelastic cylindrical panels [J]. International Journal of Non-Linear Mechanics, 2002, 37(4/5): 757-762.

[17] ou Y C, Schapery R A. Viscoelastic characterization of a nonlinear fiber-reinforced plastic [J]. Journal of Composite Materials, 1971, 5(2): 208-213.