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复合材料多层板壳在非线性弹性失稳时的本构关系

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摘 要

在研究复合材料的多层、夹层和加筋板壳的非綫性弹性和塑性失稳問題时，需要用到失稳前的应力与应变之間和失稳时的应力增量与应变增量之間的非綫性关系。复合材料在纤維断裂和基体开裂以前的物理非綫性，主要是由基体引起的。本文对非綫性弹性和处于主动塑性变形阶段的各向同性基体，采用小弹塑性变形理論，导得了应力与应变之間的各向同性非綫性关系和应力增量与应变增量之間的各向异性非綫性关系。假定纤維是綫性弹性的。用复合材料的复合定律，求得了单向增強纤維复合材料层片的应力与应变之間和应力增量与应变增量之間的非綫性关系。

一、引 言

以聚合物和金属作为基体的纤维增强复合材料，沿纤维方向拉伸和压缩时，在工程结构上常用的应变范围内，表现出良好的线性弹性，而在面内剪切和垂直于纤维方向拉伸和压缩时，尤其是使用温度升高时，则呈现比较明显的物理非线性，应变越大非线性的程度也越大。

在研究复合材料的多层、夹层和加筋板壳的非线性弹性和主动塑性变形阶段的稳定问题时，需要用到失稳前的应力与应变之间和失稳时的应力增量与应变增量之间的非线性关系。前人的工作[1][2]解决得不够完善，本文从另外的途径进行研究，在理论上比较合理。

二、基本关系

对于非线性弹性和处于主动塑性变形阶段的单向增强纤维复合材料层片的基体，用σ_i和

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ε_i表示应力强度和应变强度，用E_s和E_t表示$\sigma_i \sim \varepsilon_i$曲线的割线弹性模量和切线弹性模量，由图1可得下列关系

$$\sigma_i = E_s \varepsilon_i \quad , \delta\sigma_i = E_t \delta\varepsilon_i \qquad (2.1)$$

采用小弹塑性变形理论，对于处于平面应力状态下的层片基体，$\sigma_3 = \tau_{23} = \tau_{31} = 0$，应力偏量和应变偏量之间的关系为

$$\begin{cases} \sigma_1 - \sigma_{cp} = 2G'(\varepsilon_1 - \varepsilon_{cp}) \\ \sigma_2 - \sigma_{cp} = 2G'(\varepsilon_2 - \varepsilon_{cp}) \\ -\sigma_{cp} = 2G'(\varepsilon_3 - \varepsilon_{cp}) \\ \tau_{12} = G'\gamma_{12}, \ \sigma_{cp} = K'\varepsilon_{cp} \end{cases} \qquad (2.2)$$

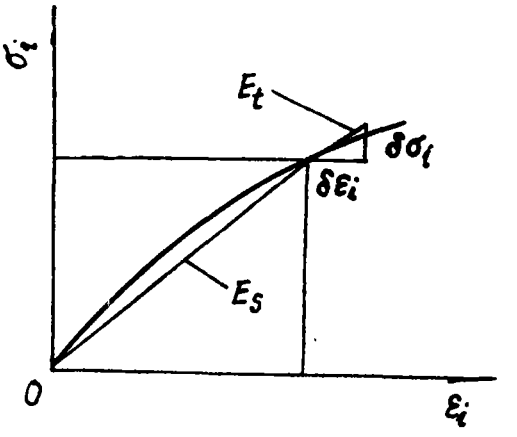

图1　单向复合材料$\sigma_i \sim \varepsilon_i$关系

其中下标1为纤维方向，下标2为层片内垂直于纤维的方向，下标3为垂直于层片的方向，σ_{cp}和ε_{cp}为平均应力和平均应变，而且

$$\begin{cases} G' = \dfrac{E_s}{2(1+\nu_m')} \ , \ K' = \dfrac{E_s}{3(1-2\nu_m')} \\ \sigma_{cp} = \dfrac{1}{3}(\sigma_1 + \sigma_2) \ , \ \varepsilon_{cp} = \dfrac{1}{3}(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) \end{cases} \qquad (2.3)$$

在(2.3)式中，G'，K'和ν_m'分别为层片的基体在主动塑性变形和非线性弹性的割线剪切模量、体积压缩模量和泊松比。在线性弹性阶段，当弹性模量为E_m时，泊松比为ν_m；当切线弹性模量$E_t = 0$时，取$\nu_m' = 0.5$；当切线弹性模量为E_t时，取

$$\nu_m' = 0.5 - \frac{E_t}{E_m}(0.5 - \nu_m) \qquad (2.4)$$

由(2.2)式可得

$$\varepsilon_3 = -\frac{\nu_m'}{1-\nu_m'}(\varepsilon_1 + \varepsilon_2) \qquad (2.5)$$

在三维应力状态下，σ_i和ε_i的表达式为

$$\begin{cases} \sigma_i = \dfrac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + 6(\tau_{12}^2 + \tau_{23}^2 + \tau_{31}^2)} \\ \varepsilon_i = \dfrac{1}{\sqrt{2}(1+\nu_m')} \sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2 + \dfrac{3}{2}(\gamma_{12}^2 + \gamma_{23}^2 + \gamma_{31}^2)} \end{cases} \qquad (2.6)$$

对于平面应力状态，由(2.6)式可得

$$\begin{cases} \sigma_i = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 + 3\tau_{12}^2} \\ \varepsilon_i = \dfrac{1}{1-\nu_m'^2} \sqrt{(1-\nu_m'+\nu_m'^2)(\varepsilon_1^2 + \varepsilon_2^2) + (-1+4\nu_m'-\nu_m'^2)\varepsilon_1\varepsilon_2 + \dfrac{3}{4}(1-\nu_m')^2\gamma_{12}^2} \end{cases} \qquad (2.7)$$

由(2.1)～(2.3)式可得基体应力和应变之间的各向同性的非线性关系

$$\sigma_1=\frac{E_s}{1-\nu_m'^2}(\varepsilon_1+\nu_m'\varepsilon_2)=\frac{1}{1-\nu_m'^2}\frac{\sigma_i}{\varepsilon_i}(\varepsilon_1+\nu_m'\varepsilon_2)$$

$$\sigma_2=\frac{E_s}{1-\nu_m'^2}(\varepsilon_2+\nu_m'\varepsilon_1)=\frac{1}{1-\nu_m'^2}\frac{\sigma_i}{\varepsilon_i}(\varepsilon_2+\nu_m'\varepsilon_1) \tag{2.8}$$

$$\tau_{12}=G'\gamma_{12}=\frac{1}{2(1+\nu_m')}\frac{\sigma_i}{\varepsilon_i}\gamma_{12}$$

和

$$\varepsilon_1=\frac{\varepsilon_i}{\sigma_i}(\sigma_1-\nu_m'\sigma_2)$$

$$\varepsilon_2=\frac{\varepsilon_i}{\sigma_i}(\sigma_2-\nu_m'\sigma_1) \tag{2.9}$$

$$\gamma_{12}=2(1+\nu_m')\frac{\varepsilon_i}{\sigma_i}\tau_{12}$$

用m表示基体，s表示割线方向的量，由(2.8)式得

$$\begin{Bmatrix}\sigma_1^m\\ \sigma_2^m\\ \tau_{12}\end{Bmatrix}=\begin{bmatrix}Q_{11}^{ms} & Q_{12}^{ms} & 0\\ Q_{12}^{ms} & Q_{22}^{ms} & 0\\ 0 & 0 & Q_{66}^{ms}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix} \tag{2.10}$$

其中

$$\begin{cases}Q_{11}^{ms}=\dfrac{E_s}{1-\nu_m'^2} & Q_{12}^{ms}=\dfrac{\nu_m'E_s}{1-\nu_m'^2}\\[2ex] Q_{22}^{ms}=\dfrac{E_s}{1-\nu_m'^2} & Q_{66}^{ms}=\dfrac{E_s}{2(1+\nu_m')}\end{cases} \tag{2.11}$$

在推导应力增量和应变增量之间的变分关系时，为了简单起见，忽略ν_m'在变分时的微小变化，假定ν_m'为常数，由(2.8)式可导得基体的应力变分与应变变分之间的关系为

$$\delta\sigma_1=\frac{E_s}{1-\nu_m'^2}(\delta\varepsilon_1+\nu_m'\delta\varepsilon_2)+\frac{\sigma_1}{\sigma_i}(E_t-E_s)\delta\varepsilon_i$$

$$\delta\sigma_2=\frac{E_s}{1-\nu_m'^2}(\delta\varepsilon_2+\nu_m'\delta\varepsilon_1)+\frac{\sigma_2}{\sigma_i}(E_t-E_s)\delta\varepsilon_i \tag{2.12}$$

$$\delta\tau_{12}=\frac{1}{2(1+\nu_m')}\delta\gamma_{12}+\frac{\tau_{12}}{\sigma_i}(E_t-E_s)\delta\varepsilon_i$$

由(2.7)第二式和(2.9)式可得

$$\delta\varepsilon_i=\frac{1}{1-\nu_m'^2}\left[\left(\frac{2-\nu_m'}{2}\frac{\sigma_1}{\sigma_i}-\frac{1-2\nu_m'}{2}\frac{\sigma_2}{\sigma_i}\right)\delta\varepsilon_1+\left(\frac{2-\nu_m'}{2}\frac{\sigma_2}{\sigma_i}\right.\right.$$

$$\left.\left.-\frac{1-2\nu_m'}{2}\frac{\sigma_1}{\sigma_i}\right)\delta\varepsilon_2\right]+\frac{3}{2(1+\nu_m')}\frac{\tau_{12}}{\sigma_i}\delta\gamma_{12} \tag{2.13}$$

将(2.13)代入(2.12)式可得基体的应力增量和应变增量之间的各向异性的非线性关系如下：

$$\delta\sigma_1=\frac{E_s}{1-\nu_m'^2}\left\{\left[1-\frac{\sigma_1}{\sigma_i}\left(1-\frac{E_t}{E_s}\right)\left(\frac{2-\nu_m'}{2}\frac{\sigma_1}{\sigma_i}-\frac{1-2\nu_m'}{2}\frac{\sigma_2}{\sigma_i}\right)\right]\delta\varepsilon_1\right.$$

$$+\left[\nu_m'-\frac{\sigma_1}{\sigma_i}\left(1-\frac{E_t}{E_s}\right)\left(\frac{2-\nu_m'}{2}\frac{\sigma_2}{\sigma_i}-\frac{1-2\nu_m'}{2}\frac{\sigma_1}{\sigma_i}\right)\right]\delta\varepsilon_2$$

$$
\left\{
\begin{aligned}
&\qquad -\frac{3(1-\nu_m')}{2}\ \frac{\sigma_1\tau_{12}}{\sigma_i^2}\left(1-\frac{E_t}{E_s}\right)\delta\gamma_{12}\Bigg\}\\
&\delta\sigma_2=\frac{E_s}{1-\nu_m'^2}\Bigg\{\left[\nu_m'-\frac{\sigma_2}{\sigma_i}\left(1-\frac{E_t}{E_s}\right)\left(\frac{2-\nu_m'}{2}\ \frac{\sigma_1}{\sigma_i}-\frac{1-2\nu_m'}{2}\ \frac{\sigma_2}{\sigma_i}\right)\right]\delta\varepsilon_1\\
&\qquad +\left[1-\frac{\sigma_2}{\sigma_i}\left(1-\frac{E_t}{E_s}\right)\left(\frac{2-\nu_m'}{2}\ \frac{\sigma_2}{\sigma_i}-\frac{1-2\nu_m'}{2}\ \frac{\sigma_1}{\sigma_i}\right)\right]\delta\varepsilon_2\\
&\qquad -\frac{3(1-\nu_m')}{2}\ \frac{\sigma_2\tau_{12}}{\sigma_i^2}\left(1-\frac{E_t}{E_s}\right)\delta\gamma_{12}\Bigg\}\\
&\delta\tau_{12}=\frac{E_s}{1-\nu_m'^2}\Bigg\{-\frac{\tau_{12}}{\sigma_i}\left(1-\frac{E_t}{E_s}\right)\Bigg[\left(\frac{2-\nu_m'}{2}\ \frac{\sigma_1}{\sigma_i}-\frac{1-2\nu_m'}{2}\ \frac{\sigma_2}{\sigma_i}\right)\delta\varepsilon_1\\
&\qquad +\left(\frac{2-\nu_m'}{2}\ \frac{\sigma_2}{\sigma_i}-\frac{1-2\nu_m'}{2}\ \frac{\sigma_1}{\sigma_i}\right)\delta\varepsilon_2\\
&\qquad +\frac{1-\nu_m'}{2}\left[1-\frac{\tau_{12}^2}{\sigma_i^2}\left(1-\frac{E_t}{E_s}\right)\frac{3}{2(1+\nu_m')}\right]\delta\gamma_{12}\Bigg\}
\end{aligned}
\right.
\tag{2.14}
$$

用 t 表示切线方向的量，由(2.14)式可得

$$
\begin{Bmatrix}\delta\sigma_1^m\\ \delta\sigma_2^m\\ \delta\tau_{12}^m\end{Bmatrix}=\begin{bmatrix}Q_{11}^{mt} & Q_{12}^{mt} & Q_{16}^{mt}\\ Q_{21}^{mt} & Q_{22}^{mt} & Q_{26}^{mt}\\ Q_{61}^{mt} & Q_{62}^{mt} & Q_{66}^{mt}\end{bmatrix}\begin{Bmatrix}\delta\varepsilon_1\\ \delta\varepsilon_2\\ \delta\gamma_{12}\end{Bmatrix}
\tag{2.15}
$$

其中

$$
\left\{
\begin{aligned}
&Q_{11}^{mt}=\frac{E_s}{1-\nu_m'^2}\left[1-\frac{\sigma_1}{\sigma_i}\left(1-\frac{E_t}{E_s}\right)\left(\frac{2-\nu_m'}{2}\ \frac{\sigma_1}{\sigma_i}-\frac{1-2\nu_m'}{2}\ \frac{\sigma_2}{\sigma_i}\right)\right]\\
&Q_{12}^{mt}=\frac{E_s}{1-\nu_m'^2}\left[\nu_m'-\frac{\sigma_1}{\sigma_i}\left(1-\frac{E_t}{E_s}\right)\left(\frac{2-\nu_m'}{2}\ \frac{\sigma_2}{\sigma_i}-\frac{1-2\nu_m'}{2}\ \frac{\sigma_1}{\sigma_i}\right)\right]\\
&Q_{21}^{mt}=\frac{E_s}{1-\nu_m^2}\left[\nu_m'-\frac{\sigma_2}{\sigma_i}\left(1-\frac{E_t}{E_s}\right)\left(\frac{2-\nu_m'}{2}\ \frac{\sigma_1}{\sigma_i}-\frac{1-2\nu_m'}{2}\ \frac{\sigma_2}{\sigma_i}\right)\right]\\
&Q_{22}^{mt}=\frac{E_s}{1-\nu_m'^2}\left[1-\frac{\sigma_2}{\sigma_i}\left(1-\frac{E_t}{E_s}\right)\left(\frac{2-\nu_m'}{2}\ \frac{\sigma_2}{\sigma_i}-\frac{1-2\nu_m'}{2}\ \frac{\sigma_1}{\sigma_i}\right)\right]\\
&Q_{16}^{mt}=-\frac{3E_s}{2(1+\nu_m')}\ \frac{\sigma_1\tau_{12}}{\sigma_i^2}\left(1-\frac{E_t}{E_s}\right)\\
&Q_{26}^{mt}=-\frac{3E_s}{2(1+\nu_m')}\ \frac{\sigma_2\tau_{12}}{\sigma_i^2}\left(1-\frac{E_t}{E_s}\right)\\
&Q_{61}^{mt}=-\frac{E_s}{1-\nu_m'^2}\ \frac{\tau_{12}}{\sigma_i}\left(1-\frac{E_t}{E_s}\right)\left(\frac{2-\nu_m'}{2}\ \frac{\sigma_1}{\sigma_i}-\frac{1-2\nu_m'}{2}\ \frac{\sigma_2}{\sigma_i}\right)\\
&Q_{62}^{mt}=-\frac{E_s}{1-\nu_m'^2}\ \frac{\tau_{12}}{\sigma_i}\left(1-\frac{E_t}{E_s}\right)\left(\frac{2-\nu_m'}{2}\ \frac{\sigma_2}{\sigma_i}-\frac{1-2\nu_m'}{2}\ \frac{\sigma_1}{\sigma_i}\right)\\
&Q_{66}^{mt}=\frac{E_s}{2(1+\nu_m')}\left[1-\frac{\tau_{12}^2}{\sigma_i^2}\left(1-\frac{E_t}{E_s}\right)\frac{3}{2(1+\nu_m')}\right]
\end{aligned}
\right.
\tag{2.16}
$$

在研究塑性力学问题时，通过采用材料不可压缩的假定，取 $\nu_m'=0.5$ 对于基体，采用不

可压缩的假定，可使上述的有关公式得到简化。由(2.3)式可得

$$G'=\frac{E_s}{3}\ ,\ K'=\infty \tag{2.17}$$

由(2.7)式可得

$$\begin{cases}\sigma_i=\sqrt{\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2+3\tau_{12}^2}\\ \varepsilon_i=\dfrac{2}{\sqrt{3}}\sqrt{\varepsilon_1^2+\varepsilon_2^2+\varepsilon_1\varepsilon_2+\dfrac{1}{4}\gamma_{12}^2}\end{cases} \tag{2.18}$$

由(2.11)式可得

$$\begin{cases}Q_{11}^{ms}=\dfrac{4}{3}E_s\ ,\ Q_{12}^{ms}=\dfrac{2}{3}E_s\\ Q_{22}^{ms}=\dfrac{4}{3}E_s\ ,\ Q_{66}^{ms}=\dfrac{1}{3}E_s\end{cases} \tag{2.19}$$

由(2.16)式可得基体的具有正交各向异性的Q_{ij}^{mt}（i，$j=1$，2，6）

$$\begin{cases}Q_{11}^{mt}=\dfrac{4}{3}E_s\left[1-\dfrac{3}{4}\dfrac{\sigma_1^2}{\sigma_i^2}\left(1-\dfrac{E_t}{E_s}\right)\right]\\ Q_{12}^{mt}=\dfrac{4}{3}E_s\left[\dfrac{1}{2}-\dfrac{3}{4}\dfrac{\sigma_1\sigma_2}{\sigma_i^2}\left(1-\dfrac{E_t}{E_s}\right)\right]\\ Q_{22}^{mt}=\dfrac{4}{3}E_s\left[1-\dfrac{3}{4}\dfrac{\sigma_2^2}{\sigma_i^2}\left(1-\dfrac{E_t}{E_s}\right)\right]\\ Q_{16}^{mt}=-E_s\dfrac{\sigma_1\tau_{12}}{\sigma_i^2}\left(1-\dfrac{E_t}{E_s}\right)\\ Q_{26}^{mt}=-E_s\dfrac{\sigma_2\tau_{12}}{\sigma_i^2}\left(1-\dfrac{E_t}{E_s}\right)\\ Q_{61}^{mt}=Q_{16}^{mt}\\ Q_{62}^{mt}=Q_{26}^{mt}\\ Q_{66}^{mt}=\dfrac{1}{3}E_s\left[1-\dfrac{\tau_{12}^2}{\sigma_i^2}\left(1-\dfrac{E_t}{E_s}\right)\right]\end{cases} \tag{2.20}$$

对于纤维增强复合材料的纤维，在应变不大（小于0.01）的情况下，具有很好的线性弹性，用f表示与纤维有关的量，可得

$$\begin{Bmatrix}\sigma_1^f\\ \sigma_2^f\\ \tau_{12}^f\end{Bmatrix}=\begin{bmatrix}Q_{11}^f & Q_{12}^f & 0\\ Q_{12}^f & Q_{22}^f & 0\\ 0 & 0 & Q_{66}^f\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix} \tag{2.21}$$

和

$$\begin{Bmatrix}\delta\sigma_1^f\\ \delta\sigma_2^f\\ \delta\tau_{12}^f\end{Bmatrix}=\begin{bmatrix}Q_{11}^f & Q_{12}^f & 0\\ Q_{12}^f & Q_{22}^f & 0\\ 0 & 0 & Q_{66}^f\end{bmatrix}\begin{Bmatrix}\delta\varepsilon_1\\ \delta\varepsilon_2\\ \delta\gamma_{12}\end{Bmatrix} \tag{2.22}$$

在(2.21)和(2.22)式中，对于各向同性的纤维（也可近似地用于各向异性的纤维）可得

$$\begin{cases} Q_{11}^{f}=\dfrac{E_f}{1-\nu_f^2}, \quad Q_{12}^{f}=\dfrac{\nu_f E_f}{1-\nu_f^2} \\ Q_{22}^{f}=\dfrac{E_f}{1-\nu_f^2}, \quad Q_{66}^{f}=G_f=\dfrac{E_s}{2(1+\nu_f)} \end{cases} \tag{2.23}$$

由基体的应力应变关系(2.10)式和纤维的应力应变关系(2.21)式可由下一节的公式求得纤维复合材料单向增强层片的应力应变关系；由基体的应力增量与应变增量间的关系(2.15)式和纤维的应力增量与应变增量间的关系(2.22)式可求得复合材料层片的应力增量与应变增量间的关系。

三、复合材料层片在非线性弹性失稳时的本构关系

单向增强纤维复合材料层片在非线性弹性失稳时，由于σ_1，σ_2，τ_{12}，σ_i和ε_1，ε_2，γ_{12}，ε_i的变化，使得失稳前的应力与应变之间和失稳时的应力增量与应变增量之间的非线性关系变得复杂了。纤维形状和纤维排列的不规则，基体中孔隙和多胶区域的存在，界面结合的不完善等，使得复合材料在失稳前和失稳时的本构关系，不可能精确地只可能近似地得到解决。由于情况的复杂性，采用以理论分析为主，再作少量而必要的实验，可使这个问题得到比较合理的解决。本文所探讨的就是这种方案的理论分析部分。

由纤维和基体的弹性模量、剪切模量、泊松比和体积百分比E_f，E_m，G_f，G_m，ν_f，ν_m和V_f，V_m，根据复合材料的复合定律[3~5]，可得单向增强层片沿纤维方向的弹性模量E_1，垂直于纤维方向的面内弹性模量E_2，剪切模量G_{12}和泊松比ν_{12}。

$$\begin{cases} E_1=E_fV_f+E_mV_m, \quad \nu_{12}=\nu_fV_f+\nu_mV_m \\ E_2=\dfrac{E_fE_m}{E_fV_m+E_mV_f}, \quad G_{12}=\left(\dfrac{V_f}{G_f}+\dfrac{1}{2}\ \dfrac{V_m}{G_m}\right)\Big/\left(V_f+\dfrac{1}{2}V_m\right) \end{cases} \tag{3.1}$$

在文献[4](2.23)式给出纤维方向的切线弹性模量

$$E_{1t}=E_fV_f+\left(\frac{d\sigma_m}{d\varepsilon_m}\right)_{\varepsilon_i}V_m \tag{3.2}$$

其中$\left(\dfrac{d\sigma_m}{d\varepsilon_m}\right)_{\varepsilon_i}$表示基体应变为$\varepsilon_i$时的切线弹性模量。

用复合定律可得非线性弹性和主动塑性变形阶段的复合材料层片的割线弹性模量和切线弹性模量。应力应变关系为

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{pmatrix}=\begin{pmatrix} Q_{11}^s & Q_{12}^s & 0 \\ Q_{12}^s & Q_{22}^s & 0 \\ 0 & 0 & Q_{66}^s \end{pmatrix}\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} \tag{3.3}$$

其中，由(3.1)式近似地可得

$$\begin{cases} Q_{11}^s=Q_{11}^sV_f+Q_{11}^{ms}V_m, \quad Q_{12}^s=(\nu_fV_f+\nu_m'V_m)Q_{22}^s \\ Q_{22}^s=\dfrac{Q_{22}^fQ_{22}^{ms}}{Q_{22}^fV_m+Q_{22}^{ms}V_f}, \quad Q_{66}^s=\dfrac{Q_{66}^fQ_{66}^{ms}\left(V_f+\dfrac{1}{2}V_m\right)}{\dfrac{1}{2}Q_{66}^fV_m+Q_{66}^{ms}V_f} \end{cases} \tag{3.4}$$

应力增量和应变增量之间的关系为

$$\begin{pmatrix} \delta\sigma_1 \\ \delta\sigma_2 \\ \delta\tau_{12} \end{pmatrix} = \begin{pmatrix} Q_{11}^t & Q_{12}^t & Q_{16}^t \\ Q_{21}^t & Q_{22}^t & Q_{26}^t \\ Q_{61}^t & Q_{62}^t & Q_{66}^t \end{pmatrix} \begin{pmatrix} \delta\varepsilon_1 \\ \delta\varepsilon_2 \\ \delta\gamma_{12} \end{pmatrix} \tag{3.5}$$

其中，由(3.1)式和(2.16)式，参照(2.20)式作简化后，近似地可得

$$\begin{cases} Q_{11}^t = Q_{11}^f V_f + Q_{11}^{mt} V_m, \quad Q_{12}^t = (\nu_f V_f + \nu_m' V_m) Q_{22}^t \approx Q_{21}^t \\ Q_{22}^t = \dfrac{Q_{22}^f Q_{22}^{mt}}{Q_{22}^f V_m + Q_{22}^{mt} V_f}, \quad Q_{66}^t = \dfrac{Q_{66}^f Q_{66}^{mt}\left(V_f + \dfrac{1}{2} V_m\right)}{\dfrac{1}{2} Q_{66}^f V_m + Q_{66}^{ms} V_f} \\ Q_{16}^t = Q_{16}^{mt} V_m, \quad Q_{26}^t = Q_{26}^{mt} V_m, \quad Q_{61}^t = Q_{61}^{mt} V_m, \quad Q_{62}^t = Q_{62}^{mt} V_m \end{cases} \tag{3.6}$$

在(3.4)和(3.6)式中，Q_{11}^s和Q_{11}^t是纤维方向的量，是纤维增强复合材料中起主要作用和十分接近线性弹性的量；Q_{22}^s和Q_{22}^t是在层片与纤维垂直方向的量，非线性比较明显，但除单向铺层情况外，对于多层板壳在正交和$\pm\theta$斜交铺层时，由于别的铺层的纤维在此方向发挥作用，因此，Q_{22}^s和Q_{22}^t起很次要的作用；Q_{66}^s和Q_{66}^t的非线性更为明显，是起比较重要作用的非线性因素；Q_{16}^t，Q_{26}^t，Q_{61}^t和Q_{62}^t也是起很次要作用的量，当$\tau_{12}=0$或$\sigma_1=\sigma_2=0$时，它们等于零。

在(3.4)和(3.6)式中，需要用到E_f，G_f，ν_f，V_f和E_m，ν_m，V_m的值，这是比较容易测定的；需要用到基体的σ_i和ε_i间的关系曲线，这可由单向增强复合材料层片在面内受剪切时测得的$\tau_{12}\sim\gamma_{12}$曲线及E_f，G_f，ν_f，ν_m，V_f，V_m等由(3.1)式第四式推算出来，这比单用基体材料所做的试件测得的数据更能反映复合材料中基体的真实情况。有了上述全部测量数据，就可求得单向增强复合材料层片在σ_1，σ_2，τ_{12}作用下的应力与应变之间和应力增量与应变增量之间的非线性关系，在此基础上，可求解复合材料多层板壳的非线性弹性或塑性失稳问题。

参 考 文 献

[1] Hahn, H. T., Nonlinear behavior of laminated composites, J. of Composite Materials, 7, 257-271 (1973).

[2] Morgan, H.S. and Jones, R. M., Buckling of rectangular cross-ply laminated plates with nonlinear stress-strain behavior, J. of Applied Mechanics, Trans. ASME, 46, 637-643 (1979).

[3] Jones, R. M., Mechanics of Composite Materials, McGraw-Hill, (1975).

[4] Agarwal, B. D. and Broutman, L. J., Analysis and Performance of Fiber Composites, J. Wiley Interscience, (1980).

[5] Tsai, S. W. and Hahn, H. T., Introduction to Composite Materials, Technomic Pub. Co. Inc., (1980).

CONSTITUTIVE RELATIONS FOR NONLINEAR ELASTIC BUCKLING OF LAMINATED PLATES AND SHELLS

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Abstract

In order to investigate the nonlinear elastic and plastic buckling problems of laminated, sandwiched and stiffened plates and shells of composite materials, the nonlinear relations between stress and strain before buckling and the nonlinear relations between stress increments and strain increments at buckling are to be established. Before breaking of fibers and cracking of matrix the nonlinear elastic behaviour of composites is mainly due to that of matrix. For the isotropic matrix under the active plastic deformation range, the small elastis-plastic deformation theory is used herein, and the nonlinear isotropic relations between stress and strain and the nonlinear anisotropic relations between stress increments and strain increments are presented .By assuming that the fibers are linear and using the law of mixture the nonlinear relatons between stress and strain, and between stress increments and strain increments of unidirectional composites are proposed.

A NEW QUADRATIC STRENGTH CRITERION FOR FIBER-REINFORCED COMPOSITES

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Abstract

This paper discucces some current strength criteria for composites. By the use of the stress tensor invariant principle a general strength criterion of the second-order power for three-dimensional unidirectional composites is propsed as following:

$$AI_1 + BI_1^2 + CI_2 + DI_2^2 + BI_1I_2 + FI_3 + GI_4 = 1 \tag{1}$$

Considering the basic failure modes of fiber-reinforced composites, a new strength criterion can be derived in the form:

$$A\sigma_X + C(\sigma_Y + \sigma_Z) + B\sigma_X^2 + D(\sigma_Y^2 + \sigma_Z^2) + (2D - G)\sigma_Y\sigma_Z + F(\tau_{XY}^2 + \tau_{XZ}^2) + G\tau_{YZ}^2 = 1 \tag{2}$$